First Semester M.Sc. in Physics Examination, May 2015
MATHEMATICAL METHODS FOR PHYSICS

Time : 3 Hours
Max. Marks : 80

Instruction: Answer all questions.

1. a) Define Linear dependence and linear independence of vectors. If \( y_1, y_2, \ldots, y_n \)
be a system of vectors such that \( y_i = \sum_{j=1}^{n} a_{ij} x_j \) where \( |a_{ij}| \neq 0 \) and \( x_1, x_2, \ldots, x_n \)
are linearly independent. Prove that \( y_1, y_2, \ldots, y_n \) are also linearly independent. \( 10 \)
b) Let \( e_1, e_2, \ldots, e_n \) be a basis of \( V \). Then for any linear operators
\( S, T \in A(V), [ST]_e = [S]_e \cdot [T]_e \)

OR

2. a) i) Prove that the eigenvectors belonging to distinct eigenvalues of a linear
transformation are linearly independent. \( 10 \)
ii) Prove that the eigenvalues of a Hermitian matrix are real.

b) Find the Fourier transform of the function \( f(t) = \begin{cases} 0 & t < -\pi \\ 1 & -\pi \leq t \leq \pi \\ 0 & \pi < t \end{cases} \) \( 5 \)

3. a) A quantity \( A(p, q, r) \) in the coordinate system \( x_{qe}^l \), such that \( A(p, q, r) B_{r}^{qs} = C_{p}^{S} \)
where \( B_{r}^{qs} \) and \( C_{p}^{S} \) are tensors. Prove that \( A(p, q, r) \) is a tensor. \( 10 \)
b) Show that \((-y, x)\) are the components of a tensor of rank 1 in two dimension. \( 5 \)

OR

4. a) Prove that the covariant derivative of \( A^p \) is a tensor. \( 10 \)
b) Express grad, div, Curl in spherical polar coordinate system. \( 5 \)

5. a) Separate the Helmholtz equation \( \left( V^2 + k^2 \right) \psi = 0 \) in spherical polar coordinates
and identify the ordinary differential equations. \( 10 \)
b) Show that Legendre’s equation has regular singularities at \( x = +1 \) and \( -1 \),
the equation is given by \( (1-x^2)y'' - 2xy' + n(n+1)y = 0 \) where \( n \) is a constant. \( 5 \)

OR

P.T.O.
6. a) Transform a second order differential equation into an integral equation.
   b) Transform the following differential equation into an integral equation

\[
\left( \frac{dx}{dt} - x \right) = 0 \text{ with } x = 1, \text{ when } t = 0.
\]

7. a) Solve the Laguerre differential equation \( xy'' + (1 - x)y' + \lambda y = 0 \) by using Frobenius’ method.
   b) Deduce \( P_0(x), P_1(x), P_2(x), P_3(x) \) from the series formula.

8. a) Prove the following recurrence relations for Bessel’s function

i) \( 2nJ_n(x) = x(J_{n+1}(x) + J_{n-1}(x)) \)

ii) \( J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x) \)
   b) Define gamma and beta functions.

9. Answer any four of the following: \((4 \times 5 = 20)\)

   a) Show that the vectors \( x_1 = (2, 1, 1), x_2 = (2, 1, 2) \) and \( x_3 = (0, 0, 1) \) are linearly independent.
   b) Prove that the eigenvalues of a unitary matrix are of modulus unity.
   c) Prove that the following is a tensor of rank 2 in 2 dimensional space

\[
\mathbf{A} = \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}.
\]
   d) Find the tensor nature of \( A_{(i, j)} \).
   e) Transform the differential equation \( \frac{d^2 y}{dx^2} + xy = 0 \) into an integral equation subject to the boundary condition \( y(0), y(b) = 0 \).
   f) Solve \( \varphi(x) = x + \frac{1}{2} \int_{-1}^{1} (t - x) \varphi(t) dt \) using Neumann series method.
   g) Prove that \( xL''_n(x) = n L_n(x) - n L_{n-1}(x) \).
   h) Show that \( \frac{d}{dx} \left\{ x^n J_n(x) \right\} = x^n J_{n-1}(x) \).
First Semester M.Sc. in Physics Examination, May 2015
CLASSICAL MECHANICS

Time: 3 Hours
Max. Marks: 80

**Instruction:** Answer all questions.

1. a) Define center of mass for a system of particles.
   b) Prove that the total angular momentum of a system of particles is conserved in the absence of applied torques. Obtain an expression for angular frequency of a charged particle moving in magnetic field.

   OR

2. a) With schematic diagrams, write down the equations of constraints in the case of a simple pendulum moving in the xy plane and a particle moving on or outside the surface of a sphere of radius a.
   b) Starting from D'Alembert's principle obtain Lagrange's equations of motion for the conservative system.

3. a) Solve the inverse Kepler problem using Binet's equation and show that the central force may be in the form of universal law of gravitation.
   b) State Hamilton's least action principle. Derive Hamilton's equations from the variational principle.

   OR

4. a) Explain the term Canonical transformation. Obtain simplectic condition for a canonical transformation.
   b) Express the total time derivative \( \frac{dA}{dt} \) of a function \( A(p_k, q_k, t) \) in terms of the Poisson bracket of \( A \) with the Hamiltonian \( H \).

5. a) Describe Hamilton-Jacobi method for solving the equation for a one dimensional linear harmonic oscillator and obtain the solution.
   b) State and prove parallel axis theorem.

   OR

P.T.O.
6. a) What are the principal moments of inertia of a rigid body? Classify rigid bodies based on their principal moments of inertia.  
   b) Setup the Euler equations of motion for a rigid body.  

7. a) Discuss the simultaneity in relativity.  
   b) Obtain the Lorentz covariant form of Newton's second law of motion. Determine the 4th component of the 4-force along the world line of the particle assuming that the rest mass of the particle is finite, real and non-zero it does not vary along the world line.  

OR

8. a) Obtain the expression for relativistic energy of a particle of rest mass \( m \).  
   b) What is the difference between inertial mass and gravitational mass? Describe the Eötvös experiment to demonstrate the identity between inertial mass and gravitational mass.  

9. Answer any four of the following:  
   (4x5=20)
   a) Verify whether \( \vec{F} = (3x^2 - 3y^2) \hat{i} + (4x - 6xy) \hat{j} \) is a conservative force or not.  
   b) Write down the Lagrange's equations, when the Lagrangian function is  
      \[ L = q_k \dot{q}_k - (1 - c_k^2)^{1/2}, \]
   c) One of the attempts at combining the two sets of Hamilton's equations into tries to take \( q \) and \( p \) as forming a complex quantity. Show directly from Hamilton's equations of motion that for a system of one degree of freedom the transformation \( Q = q + ip \), \( P = Q^* \) is not canonical if the Hamiltonian is left unaltered. Can you find another set of coordinates \( Q', P' \) that are related to \( Q, P \) by a change of scale only and that are canonical?  
   d) Using the fundamental Poisson brackets show that the following transformation is canonical  
      \[ Q = \sqrt{2} q e^{i} \cos p \text{ and } P = \sqrt{2} q e^{i} \sin p. \]
e) A rigid body in motion has $\vec{\omega} = 2\hat{k}$ and the moment of inertia tensor is

$$
I = \begin{pmatrix}
1 & 2 & 0 \\
2 & 4 & 0 \\
0 & 0 & 6
\end{pmatrix}
$$

Calculate the angular momentum vector $\vec{L}$ of the rigid body.

f) If $T$ be the kinetic energy, $G$ the external torque about the instantaneous axis and $\vec{\omega}$ the resultant angular velocity, show that $\frac{dT}{dt} = G \vec{\omega}$.

g) At what speed does a matter stick move if its length is observed to shrink to 0.75 m.

h) A body of mass 10 $m_e$ is moving with a velocity $\vec{v} = \frac{c}{\sqrt{2}} \hat{i} + \frac{c}{\sqrt{3}} \hat{j}$, calculate all components of its four momentum.
First Semester M.Sc. in Physics Examination, May 2015
ATOMIC AND MOLECULAR PHYSICS

Time : 3 Hours

Max. Marks : 80

Instruction: Answer all questions.

1. a) Derive an expression for the rotational energy of a diatomic molecule using rigid rotator.  
   b) Explain vibrational progression and sequence in the electronic spectrum of a diatomic molecule with a neat diagram.
   OR
2. a) Obtain an expression for Franck-Condon factor and explain its significance.  
   b) Write a note on the formation of band head in rotational band spectra for $B'_v < B''_v$ using Fortrat parabola.
3. a) Describe the parameters of a molecular structure.  
   b) Write the applications of lattice energy.
   OR
4. a) Explain the formation of $\sigma$-bond and $\pi$-bond.  
   b) Discuss sp-hybridization along with its characteristics.
5. a) Explain the processes of induced absorption, spontaneous emission and stimulated emission in Lasers using Einstein's coefficients.  
   b) Write a note on coherence in Lasers.
   OR
6. a) Explain the construction and working of Ruby Laser.  
   b) Write the applications of Holography techniques.
7. a) Define Harmonic generation. How second harmonic generation can be observed experimentally?  
   b) Explain the environmental applications of Laser Raman Spectroscopy.
   OR
8. a) What are the mode locking of Lasers? Discuss with examples.  
   b) Write a note on Q-switched Lasers.

P.T.O.
9. Answer any four of the following: (4x5=20)

a) Arrive at an expression for rotational level with maximum intensity using Boltzmann distribution law.

b) Discuss the intensity distribution in the vibrational structure for \( r''_0 < r'_0 \).

c) Write a note on Ionic bond.

d) Write a note on close packing structure.

e) What do you mean by pumping in Lasers? What are its types?

f) Discuss the construction of \( \text{CO}_2 \) laser along with the neat diagram.

g) Why Stokes' lines are more intense than anti-stokes' lines?

h) What is homogeneous and inhomogeneous broadening?
First Semester M.Sc. in Physics Examination, May 2015
SOLID STATE PHYSICS AND ELECTRONIC DEVICES

Time : 3 Hours  Max. Marks : 80

Instruction: Answer all questions.

1. a) What is reciprocal lattice? Derive expressions for reciprocal lattice vectors. 10  
   b) Distinguish between first and second Brillouin zones. 5  
   OR
2. a) What is a Fermi surface? Discuss the experimental method of determining Fermi surface of a metal. 10  
   b) Discuss a method of determining Fermi surface of a metal experimentally. 5
3. a) What is Kondo effect? Describe how magnetic impurities contribute to electrical properties of metals. 10  
   b) Write a note on Cyclotron resonance. 5  
   OR
4. a) Discuss electrical conductivity of metals at high frequency and obtain an expression for dielectric constant. 10  
   b) Describe Umklapp process with a neat diagram. 5
5. a) Arrive at an expression for carrier concentration in an intrinsic semiconductor. 10  
   b) Write a note on Fermi level in an extrinsic semiconductor. 5  
   OR
6. a) Obtain an expression for Fermi energy in case of an extrinsic semiconductor. 10  
   b) Discuss Franck-Condon principle with a diagram. 5
7. a) Discuss the formation of space charge region in p-n junction diode and obtain an expression for the width of the region. 10  
   b) Write a note on diffusion capacitance associated with a p-n junction. 5  
   OR
8. a) With a neat diagram, discuss the construction and working of a JFET. 10  
   b) Write a note on $\alpha$-cutoff frequency in a transistor. 5

P.T.O.
9. Answer any four of the following: \( (4 \times 5 = 20) \)

a) Show that the reciprocal lattice of a bcc is fcc.

b) Discuss the band structure of an electron in a 2-d lattice and show that the Fermi surface lies inside the first Brillouin zone.

c) Calculate the plasma frequency of copper and its dielectric constant for wavelength 540 nm. Given: ‘\( n \)’ of copper = \( 2.66 \times 10^{20} \text{m}^{-3} \).

d) Calculate the Fermi level and the conductivity at 300 K for germanium crystal containing \( 5 \times 10^{22} \) arsenic atoms per cubic meter.

e) Resistivity of pure silicon is 2300 \( \Omega \) m and the mobilities of electrons and holes in it are 0.125 \( \text{m}^2\text{V}^{-1}\text{s}^{-1} \) and 0.046 \( \text{m}^2\text{V}^{-1}\text{s}^{-1} \) respectively. Find the electron and hole concentrations and the resistivity of a specimen of silicon doped with \( 10^{19} \) atoms of phosphorous per cubic meter.

f) Write a note on Schotky effect.

g) Find the \( \alpha \)-cutoff frequency of a npn transistor with base width 1 \( \mu \text{m} \). Given : \( \tau_p = 5 \times 10^{-7} \text{s} \) and \( D_p = 20 \text{ cm}^2 \text{s}^{-1} \).

h) A UJT has \( R_{B1} = 900 \text{ k}\Omega \) and \( R_{B2} = 100 \text{ k}\Omega \). Calculate the voltage drop across \( R_{B1} \). Given \( V_{B1B2} = 15 \text{ V} \).