MP 1.1

First Semester M.Sc. in Physics Examination, May 2015
MATHEMATICAL METHODS FOR PHYSICS

Time : 3 Hours Max. Marks : 80

Instruction : *Answer **all** questions.*

1. a) Define Linear dependence and linear independence of vectors. If $y_1, y_2, \ldots y_n$ be a system of vectors such that $y_i = \sum_{j=1}^{n} a_{ij} x_j$ where $|a_{ij}| \neq 0$ and $x_1, x_2, \ldots x_n$ are linearly independent. Prove that $y_1, y_2, \ldots y_n$ are also linearly independent. **10**

   b) Let $e_1, e_2, \ldots e_n$ be a basis of V. Then for any linear operators $S, T \in A(V)$, $[ST]_e = [S]_e [T]_e$ **5**

OR

2. a) i) Prove that the eigenvectors belonging to distinct eigenvalues of a linear transformation are linearly independent. **10**

   ii) Prove that the eigenvalues of a Hermitian matrix are real.

   b) Find the Fourier transform of the function $f(t) = \begin{cases} 0 & t < -\pi \\ 1 & -\pi \leq t \leq \pi \\ 0 & \pi < t \end{cases}$ **5**

3. a) A quantity A(p, q, r) in the coordinate system x^i_{qs}, such that $A(p, q, r)\, B^{qs}_r = C^s_P$ where B^{qs}_r and C^s_P are tensors. Prove that A(p, q, r) is a tensor. **10**

   b) Show that (–y, x) are the components of a tensor of rank 1 in two dimension. **5**

OR

4. a) Prove that the covariant derivative of A^P is a tensor. **10**

   b) Express grad, div, Curl in spherical polar coordinate system. **5**

5. a) Separate the Helmholtz equation $(\nabla^2 + k^2)\psi = 0$ in spherical polar coordinates and identify the ordinary differential equations. **10**

   b) Show that Legendre's equation has regular singularities at $x = +1$ and -1, the equation is given by $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ where n is a constant. **5**

OR

P.T.O.

6. a) Transform a second order differential equation into an integral equation. **10**

b) Transform the following differential equation into an integral equation

$\left(\frac{dx}{dt} - x\right) = 0$ with $x = 1$, when $t = 0$. **5**

7. a) Solve the Laguerre differential equation $xy'' + (1-x)y' + \lambda y = 0$ by using Frobenius' method. **10**

b) Deduce $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$ from the series formula. **5**

OR

8. a) Prove the following recurrence relations for Bessel's function **10**

i) $2nJ_n(x) = x\{J_{n+1}(x) + J_{n-1}(x)\}$ ii) $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$

b) Define gamma and beta functions. **5**

9. Answer **any four** of the following : **(4×5=20)**

a) Show that the vectors $x_1 = (2, 1, 1)$, $x_2 = (2, 1, 2)$ and $x_3 = (0, 0, 1)$ are linearly independent.

b) Prove that the eigenvalues of a unitary matrix are of modulus unity.

c) Prove that the following is a tensor of rank 2 in 2 dimensional space

$$\overline{A} = \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}.$$

d) Find the tensor nature of $A_{(i,j)}$.

e) Transform the differential equation $\frac{d^2y}{dx^2} + xy = 0$ into an integral equation subject to the boundary condition $y(0), y(b) = 0$.

f) Solve $\varphi(x) = x + \frac{1}{2}\int_{-1}^{+1}(t-x)\,\varphi(t)dt$ using Neumann series method.

g) Prove that $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$.

h) Show that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$.

First Semester M.Sc. in Physics Examination, May 2015
CLASSICAL MECHANICS

Time : 3 Hours Max. Marks : 80

Instruction : *Answer **all** questions.*

1. a) Define center of mass for a system of particles. **5**

   b) Prove that the total angular momentum of a system of particles is conserved in the absence of applied torques. Obtain an expression for angular frequency of a charged particle moving in magnetic field. **10**

OR

2. a) With schematic diagrams, write down the equations of constraints in the case of a simple pendulum moving in the xy plane and a particle moving on or outside the surface of a sphere of radius a. **5**

   b) Starting from D'Alembert's principle obtain Lagrange's equations of motion for the conservative system. **10**

3. a) Solve the inverse Kepler problem using Binet's equation and show that the central force may be in the form of universal law of gravitation. **5**

   b) State Hamilton's least action principle. Derive Hamilton's equations from the variational principle. **10**

OR

4. a) Explain the term Canonical transformation. Obtain simpletic condition for a canonical transformation. **10**

   b) Express the total time derivative $\frac{dA}{dt}$ of a function $A(p_k, q_k, t)$ in terms of the Poisson bracket of A with the Hamiltonian H. **5**

5. a) Describe Hamilton-Jacobi method for solving the equation for a one dimensional linear harmonic oscillator and obtain the solution. **10**

   b) State and prove parallel axis theorem. **5**

OR

6. a) What are the principal moments of inertia of a rigid body ? Classify rigid bodies based on their principal moments of inertia. **5**

   b) Setup the Euler equations of motion for a rigid body. **10**

7. a) Discuss the simultaneity in relativity. **5**

   b) Obtain the Lorentz covariant form of Newton's second law of motion. Determine the 4th component of the 4-force along the world line of the particle assuming that the rest mass of the particle is finite, real and non-zero it does not vary along the world line. **10**

OR

8. a) Obtain the expression for relativistic energy of a particle of rest mass m. **5**

   b) What is the difference between inertial mass and gravitational mass ? Describe the Eötvös experiment to demonstrate the identity between inertial mass and gravitational mass. **10**

9. Answer **any four** of the following : **(4×5=20)**

   a) Verify whether $\vec{F} = (3x^2 - 3y^2)\,\hat{i} + (4x - 6xy)\,\hat{j}$ is a conservative force or not.

   b) Write down the Lagrange's equations, when the Lagrangian function is

   $L = \dot{q}_k q_k - (1 - \dot{q}_k^2)^{-1/2}$,

   c) One of the attempts at combining the two sets of Hamilton's equations into tries to take q and p as forming a complex quantity. Show directly from Hamilton's equations of motion that for a system of one degree of freedom the transformation $Q = q + ip$, $P = Q^*$ is not canonical if the Hamiltonian is left unaltered. Can you find another set of coordinates Q', P' that are related to Q, P by a change of scale only and that are canonical ?

   d) Using the fundamental Poisson brackets show that the following transformation is canonical

   $Q = \sqrt{2qe^t}\cos p$ and $P = \sqrt{2qe^t}\sin p$.

e) A rigid body in motion has $\vec{\omega} = 2\hat{k}$ and the moment of inertia tensor is

$$I = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$. Calculate the angular momentum vector $\vec{L}$ of the rigid body.

f) If T be the kinetic energy, G the external torque about the instantaneous axis and $\vec{\omega}$ the resultant angular velocity, show that $\frac{dT}{dt} = G\vec{\omega}$.

g) At what speed does a matter stick move if its length is observed to shrink to 0.75 m.

h) A body of mass 10 m_e is moving with a velocity $\vec{v} = \frac{c}{\sqrt{2}}\hat{i} + \frac{c}{\sqrt{3}}\hat{j}$, calculate all components of its four momentum.

MP 1.3

First Semester M.Sc. in Physics Examination, May 2015
ATOMIC AND MOLECULAR PHYSICS

Time : 3 Hours Max. Marks : 80

Instruction : *Answer **all** questions.*

1. a) Derive an expression for the rotational energy of a diatomic molecule using rigid rotator. **10**

   b) Explain vibrational progression and sequence in the electronic spectrum of a diatomic molecule with a neat diagram. **5**

OR

2. a) Obtain an expression for Franck-Condon factor and explain its significance. **10**

   b) Write a note on the formation of band head in rotational band spectra for $B'_v < B''_v$ using Fortrat parabola. **5**

3. a) Describe the parameters of a molecular structure. **10**

   b) Write the applications of lattice energy. **5**

OR

4. a) Explain the formation of σ-bond and π-bond. **10**

   b) Discuss sp-hybridization along with its characteristics. **5**

5. a) Explain the processes of induced absorption, spontaneous emission and stimulated emission in Lasers using Einstein's coefficients. **10**

   b) Write a note on coherence in Lasers. **5**

OR

6. a) Explain the construction and working of Ruby Laser. **10**

   b) Write the applications of Holography techniques. **5**

7. a) Define Harmonic generation. How second harmonic generation can be observed experimentally ? **10**

   b) Explain the environmental applications of Laser Raman Spectroscopy. **5**

OR

8. a) What are the mode locking of Lasers ? Discuss with examples. **10**

   b) Write a note on Q-switched Lasers. **5**

P.T.O.

9. Answer **any four** of the following : **(4×5=20)**

a) Arrive at an expression for rotational level with maximum intensity using Boltzmann distribution law.

b) Discuss the intensity distribution in the vibrational structure for $r_e'' < r_e'$.

c) Write a note on Ionic bond.

d) Write a note on close packing structure.

e) What do you mean by pumping in Lasers ? What are its types ?

f) Discuss the construction of CO_2 laser along with the neat diagram.

g) Why Stokes' lines are more intense than anti-stokes' lines ?

h) What is homogeneous and inhomogeneous broadening ?

MP 1.4

First Semester M.Sc. in Physics Examination, May 2015
SOLID STATE PHYSICS AND ELECTRONIC DEVICES

Time : 3 Hours

Max. Marks : 80

Instruction : *Answer **all** questions.*

1. a) What is reciprocal lattice ? Derive expressions for reciprocal lattice vectors. **10**

   b) Distinguish between first and second Brillouin zones. **5**

OR

2. a) What is a Fermi surface ? Discuss the experimental method of determining Fermi surface of a metal. **10**

   b) Discuss a method of determining Fermi surface of a metal experimentally. **5**

3. a) What is Kondo effect ? Describe how magnetic impurities contribute to electrical properties of metals. **10**

   b) Write a note on Cyclotron resonance. **5**

OR

4. a) Discuss electrical conductivity of metals at high frequency and obtain an expression for dielectric constant. **10**

   b) Describe Umklapp process with a neat diagram. **5**

5. a) Arrive at an expression for carrier concentration in an intrinsic semiconductor. **10**

   b) Write a note on Fermi level in an extrinsic semiconductor. **5**

OR

6. a) Obtain an expression for Fermi energy bin case of an extrinsic semiconductor. **10**

   b) Discuss Franck-Condon principle with a diagram. **5**

7. a) Discuss the formation of space charge region in p-n junction diode and obtain an expression for the width of the region. **10**

   b) Write a note on diffusion capacitance associated with a p-n junction. **5**

OR

8. a) With a neat diagram, discuss the construction and working of a JFET. **10**

   b) Write a note on α-cutoff frequency in a transistor. **5**

P.T.O.

9. Answer **any four** of the following : **(4×5=20)**

a) Show that the reciprocal lattice of a bcc is fcc.

b) Discuss the band structure of an electron in a 2-d lattice and show that the Fermi surface lies inside the first Brillouin zone.

c) Calculate the plasma frequency of copper and its dielectric constant for wavelength 540 nm. Given : 'n' of copper = $2.66 \times 10^{28}/m^3$.

d) Calculate the Fermi level and the conductivity at 300 K for germanium crystal containing 5×10^{22} arsenic atoms per cubic meter.

e) Resistivity of pure silicon is 2300 Ω m and the mobilities of electrons and holes in it are 0.125 $m^2V^{-1}s^{-1}$ and 0.046 $m^2V^{-1}s^{-1}$ respectively. Find the electron and hole concentrations and the resistivity of a specimen of silicon doped with 10^{19} atoms of phosphorous per cubic meter.

f) Write a note on Schotky effect.

g) Find the α-cutoff frequency of a pnp transistor with base width 1 μm. Given : $\tau_p = 5 \times 10^{-7}$ s and $D_p = 20\ cm^2\ s^{-1}$.

h) A UJT has $R_{B_1} = 900\ k\Omega$ and $R_{B_2} = 100\ k\Omega$. Calculate the voltage drop across R_{B_1}. Given $V_{B_1B_2} = 15$ V.