KARNATAKA STATE OPEN UNIVERSITY
Department of Studies in Mathematics
Mukthagangotri, Mysuru – 570 006

Date:20.08.2021

Third Semester Internal Assessment
(For July-2019-20 and January 2019-20)

Instructions:

1. Answer all the questions from each of the following courses. Each questions carry 5 marks.
2. Write assignment of each course separately using A4 sheets (one side).
3. The assignment of each course must be enclosed with covering sheet.
4. The assignment should be submitted to the department address.
   The Lecturer, Department of Studies in Mathematics, Vijnan Bhavan, Karnataka State Open University, Mysuru-570006.
5. The assignment must be submitted on or before **22nd of September 2021**.

Course: Math 3.1: Topology

1. Define Base for the topology. Let (X,τ) be a topological space. Prove that a subfamily $\mathcal{B}$ of τ is a base for τ if and only if $U \in \tau$ and $x \in U$ implies there is a B in $\mathcal{B}$ such that $x \in B \subseteq U$.
2. Prove that a bijective map $f:(X,\tau) \to (Y,v)$ is a homeomorphism if and only if $f(\bar{A}) = \overline{f(A)}$, for all $A \subseteq X$.
3. Define a compact space. Prove that a continuous image of a compact space is compact.
4. State and prove Urysohn's Metrization theorem.

Course: Math 3.2: Measure and integration

1. Let A be an algebra of X and $\{A_i\}$ a sequence of sets in A. Then prove that there is a sequence $\{B_i\}$ of sets in A such that $B_n \cap B_m = \emptyset$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$.
2. If f is a bounded function defined on a measurable set E with $mE < \infty$. Then prove that f is measurable if and only if, $inf_{\psi \geq f} \int \psi = sup_{\phi \leq f} \int \phi$.
3. State and prove Lebesgue convergence theorem for integrable functions.
4. If f is absolutely continuous on $[a,b]$ and $f'(x) = 0$ almost everywhere then prove that f is constant.

Course: Math 3.3: Functional Analysis

1. Define limit point of a set, closure of a set and dense set. If A is a finite in a metric space (X, d), then show that A', complement of A is open set.
2. If (X, d) is a complete metric space and if $T: X \to X$ is a contraction mapping then show that T has a unique fixed point.
3. Prove that, every sequentially compact metric space is both complete and totally bounded.
4. State Hahn- Banach theorem for a normed linear space. Prove the theorem by making use of Hahn- Banach theorem for a complex linear space.

Course: Math 3.4: Mathematical Modeling

1. Explain the techniques and characteristics of Mathematical modeling.
2. Explain linear population growth model. Find the expression for doubling / half- life period.
3. Show that the force required to make a particle of mass ' m' move in a circular orbit of radius with velocity 'v' is $\frac{mv^2}{a}$ directed towards the center.
4. Describe a partial differential equation model for birth-death-immigration process.

Course: Math 3.5: Computer Programming

1. Explain primary memory its properties and its types.
2. Explain the method for developing the algorithm. Illustrate the method to write an algorithm to compute the area of a circle.
3. Describe in detail the basic structure of C- programing.
4. What is a bisection method? Write an algorithm and C program for the bisection method.

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